CONTROLLABILITY OF LINEAR DYNAMICAL SYSTEMS

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In recent decades modern control theory of linear dynamical systems has been the subject of considerable interest of many research scientists. It has been motivated, on the one hand, by the wide range of applications of linear models in various areas of science and engineering and, on the other hand, by the difficult and stimulating theoretical problems posed by such systems.

At the end of the sixties, the state space theory of linear control systems both for time-invariant and time-varying dynamical systems had essentially been worked out. The basic concepts of controllability and the weaker notions of stabilizability and detectability play an essential, fundamental role in the solutions of many important different optimal control problems.

Systematic study of controllability was started at the beginning of sixties, when the theory of controllability and based on the description in the form of state space for both time-invariant and time-varying linear control systems was worked out. Many dynamical systems are such that the control does not affect the complete state of the dynamical system but only a part of it. Therefore, it is very important to determine whether or not control of the complete state of the dynamical system is possible.

Roughly speaking, controllability generally means, that it is possible to steer dynamical system from an arbitrary initial state to an arbitrary final state using the set of admissible controls. Controllability is also strongly connected with the theory of minimal realisation of linear time-invariant control systems. Moreover, it should be pointed out that there exists a formal duality between the concepts of controllability and observability.

In the literature there are many different definitions and conditions of controllability which depend on the type of dynamical control system (see e.g., monographs and papers [1-6] for extensive lists of publications). The main purpose of this article is to present a compact review over the existing controllability results mainly for linear continuous-time finite-dimensional and infinite-dimensional stationary control systems.

It should be pointed out that for linear control systems controllability conditions have pure algebraic forms and are rather easily computable. These conditions require verification location of the roots of a characteristic polynomial and of the rank conditions for suitable defined constant controllability matrices.

The paper is organized as follows:

Section 2 contains systems descriptions and fundamental controllability results concerning the most popular linear continuous-time finite-dimensional control models with constant coefficients. Special attention is paid for constrained controlability problems, output controllability and controllability after the introducing of sampling. The relationships between controllability of dynamical system and its stabilizability are also pointed out. Finally, minimum energy control problem for controllable systems is discussed.

Section 3 presents mathematical model of linear infinite-dimensional, stationary control system. Next, fundamental definitions of controllability are given. Moreover, using the results taken from the theory of linear unbounded operators and semigroups of linear operators necessary and sufficient conditions for different kinds of controllability are presented. As an illustrative examples approximate controllability problems for linear distributed parameter systems are investigated.

Finally, in Section 4 concluding remarks and comments concerning possible extensions are presented.

Since the article should be limited to a reasonable size, it is impossible to give a full survey on the subject. In consequence, only selected fundamental results without proofs are presented.